Elementary Inflation Theory

What follows is a highly simplified summary of the monetary theory of price inflation. For a somewhat more rigorous presentation, see McCulloch, J. Huston. *Money and Inflation: A Monetarist Approach.* New York. Academic Press. 1975

In order to express the relationship of the quantity of money in an economy with price levels and the total quantity of goods and services in a single year, we must construct a price index P in order to allow for a general change of prices for goods. (Services will be considered just a particular kind of good.) Let us suppose that we want to relate the general price level in year i with the prices in year j. If the prices in going from one year to the other were all to be multiplied by a factor m , we would want the price index connecting the two years to be $P_{ij} = m$. Next, let us make the following definitions.

- Q_k^i = the quantity of a good of type k sold and bought in year i
- P_k^i = the price of a good of type k sold and bought in year i

In constructing the price index we want to create a kind of average price in a particular year divided by an average price in a prior year, where the contribution of the price of a particular good is weighted by the number of the good that is sold and bought. Because we do not want differences in the number of goods sold in the two years to distort the price index, we will construct the average price in both years using weights calculated from the quantity of goods only in the initial year i. The weight for good type k is then given by

$$w_k \equiv \frac{Q_k^i}{\sum_m Q_m^i} = \frac{Q_k^i}{Q} \tag{1}$$

These are the weights that will be used in calculating the average prices of both years.

$$\mathcal{P}_i = \sum_k w_k P_k^i \qquad \mathcal{P}_j = \sum_k w_k P_k^j \tag{2}$$

The price index connecting an initial year i to a year j is finally defined by

$$P_{ij} \equiv \frac{\mathcal{P}_j}{\mathcal{P}_i} = \frac{\sum_m Q_m^i P_m^j}{\sum_k Q_k^i P_k^i} \tag{3}$$

All price indices are constructed in this fashion.

Using a price index we can compare an amount of money in one year with an amount of money with equivalent purchasing power in another year. Let us consider the year i to be the base or standard year with the dollars of that year being designated *constant dollars.* An amount of money M in year j, designated nominal or current dollars, is related to an equivalent constant dollar amount m by

$$M = mP_{ij} \quad , \qquad m = \frac{M}{P_{ij}} \tag{4}$$

For the balance of this paper we will denote dollar amounts in nominal or current dollars with capital letters, and constant dollar amounts expressed with lowercase letters.

Now suppose that in any given year, the current dollar value of all the transactions for that year is T. This is the GDP. If the total amount of money in the economic system in current dollars is M, and if the average number of times a current dollar is used in a transaction in the year is V, then by definition

$$MV = T \tag{5}$$

or in constant dollars, $t = T/P_{ij} = T/P$ so that

$$MV = Pt \tag{6}$$

Either equation 5 or 6 is regarded as the definition of the quantity V , which is called the *velocity of money.*

We are now in a position to derive an equation for the inflation rate using equation (6). All of the quantities in that equation can be considered to be functions of time that vary from year to year. So long as the fractional change of any of them in a single year is much less than 1. we can approximate the change of any of them with their differentials. Taking differentials in (6), we obtain

$$dP = d\left(\frac{MV}{t}\right) = \frac{VdM}{t} + \frac{MdV}{t} - \frac{MV}{t^2}dt \tag{7}$$

If we now divide this last equation with $P = MV/t$, we finally obtain

$$\frac{dP}{P} = \frac{t}{MV}\left[\frac{VdM}{t} + \frac{MdV}{t} - \frac{MV}{t^2}dt\right] \tag{8}$$

$$= \frac{dM}{M} + \frac{dV}{V} - \frac{dt}{t} \tag{9}$$

This last equation gives us the fractional inflation (if positive) or deflation (if negative) rate dP/P in terms of the fractional changes in the money supply dM/M, the velocity of money dV/V, and the GDP dt/t.